Michael Redhead

Significance of Relativity Theory.

~~Philosophical Consequences of the Theory of Relativity~~

Introduction and Overview

There are two parts to Einstein's relativity theory, the special theory published in 1905 and the general theory published in its final mathematical form in 1916. 1915. The special theory is a direct development of the Galilean relativity principle in classical Newtonian mechanics. This principle affirms that Newton's Laws of motion hold in all inertial frames not just when the motion is described relative to a reference frame at rest in absolute space (see SPACE), but also relative to any reference frame in uniform translational motion relative to absolute space. The class of frames relative to which Newton's Laws of Motion are valid are referred to as inertial frames. It follows that no mechanical experiment can tell us which frame is at absolute rest, only the relative motion of inertial frames is observable. The Galilean relativity principle does not deal with accelerated motion, and also it does not ~~for~~ deal with electromagnetic phenomena, in particular the propagation of light waves as governed by Maxwell's equations (see ELECTRODYNAMICS). Einstein's special theory of relativity reformulated the mathematical transformations for space and time coordinates between inertial reference frames, replacing the Galilean transformations by the so-called Lorentz transformations (derived previously but discussed in an entirely different way by H. A.

EINSTEIN

Lorentz in 1904 — der Einstein — EINSTEIN) on such a way that electromagnetism satisfied the relativity principle. But the classical laws of mechanics no longer did so. So Einstein next reformulated the Newton's laws of mechanics so as to make them conform to his new relativity principle ~~based on the Lorentz transformations as opposed to the Galilean transformations~~ Wat Galilean relativity stated intervals the simultaneity of events and temporal durations did not depend on the inertial frame, although of course velocities did. In Einstein's relativity the first three ~~mentioned~~ now become relativized, to frame-dependent or 'relativized' as he may express it, while for the fourth, viz. velocity there exists an upper velocity, that of light in vacuo, whose magnitude is invariant, i.e. the same for all inertial frames. Einstein Moreover in Einstein's new mechanics inertial mass m becomes a relative notion and is associated with Einstein the famous equation $E = mc^2$, $m = E/c^2$ which says forms of energy E#

In the grand theory Einstein ~~sought~~ sought to extend the relativity principle to accelerated motions of the reference frame, by employing an generalized principle which claimed it was possible to distinguish observationally between the presence of a gravitational field, and the acceleration of a reference frame. Einstein has elevated

into a fundamental principle, to posses
but apparently accidental, numerical
equality of the inertial and gravitational
mass of a body (which accounts for
the fact that bodies fall with the
same acceleration in a gravitational field,
independent of their material nature)
By extending the discussion to
gravitational fields which exist not
locally, but are globally, transformed
away, by a change of reference
frame, Einstein was led to a
new theory of gravitation modifying
Newton's theory of gravitation, but could
explain a number of observed phenomena
in which the Newtonian theory was
inadequate. Einstein's theory has had
important repercussions for philosophical
views on the nature of space
and time, and their relation to
those of causality and cosmology
which are still the subject of ongoing
debate.

5 1/2 pages
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Einstein
I It can be argued that (1) also represents the maximum
speed with which any causal process can
be propagated.

Einstein
II Reciprocally inertial mass can be understood as
a form of equivalent to a corresponding energy mc^2.

Einstein
III This involved a law (Einstein's field equations) relating the distributions of matter

associated with

in spacetime to ~~the~~ geometrical features
of spacetime ~~that are related to~~ its curvature
considered as a four-dimensional manifold
(or SPACETIME). The path of an ~~body~~ (uncharged spinless)
particle moving
freely
in the curved spacetime ~~followed~~
was a geodesic (the generalization (analogue)
a curved manifold of a straight line
in a flat manifold).

(4) 1. The Relativity of ~~Simultaneity~~ Lorentz Transformations

(6) 2. The relativity of ~~length~~ simultaneity, length, and ~~simultaneity~~ etc.

(5) 3. Causality and the conventionality of simultaneity.

(4) 4. General Covariance.

(~~4~~) (3) 5. Determinism and ~~Indeterminism~~ Substantivalism in General Relativity.

(3) 6. Mach's Principle

(2) 7. Super substantivalism

spatial
↓ the Lorentz of the K-frame.

1. The Lorentz Transformations

Consider two inertial reference frames K and K', where K' moves along the positive X-axis with velocity v as viewed from the K-frame. Let ordinary Cartesian spatial coordinates (x, y, z) and a time coordinate t be used to locate some arbitrary event relative to the K-frame, and similarly use x', y', z' and t' to locate the event relative to the K'-frame. Suppose the spatial axes are parallel in the two frames and the origins of the frames coincide at time t = t' = 0, then the classical Galilean transformations read.

$$\left.\begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned}\right\} \qquad (1)$$

The corresponding Lorentz transformations of special relativity are

$$\left.\begin{aligned} x' &= \frac{1}{\sqrt{1-v^2/c^2}}\,(x - vt) \\ y' &= y \\ z' &= z \\ t' &= \frac{1}{\sqrt{1-v^2/c^2}}\left(t - \frac{vx}{c^2}\right) \end{aligned}\right\} \qquad (2)$$

where c is the (magnitude of the) velocity of light in vacuo. For velocities small compared with c the Lorentz transformations are approximately the same as the Galilean transformations (but see Section 2 below).

& Einstein

Special Principle of Relativity

2. The Invariance Principle, that the laws of nature are invariant (take the same form) in all inertial reference frames

1. 2.
From (1) and (2) Einstein expressed

Einstein II

Einstein worked also the homogeneous of space and time to justify the Lorentz transformation equations and a Reciprocal Principle stating essentially that the transformations depend only on the relative velocity of the frames.

II

By invoking various other 'homogeneity' and 'isotropy' properties of space and time Einstein proceeded to derive the Lorentz transformation equations.
Since Einstein's 1905 paper there have been many other derivations of the Lorentz transformations from various combinations of plausible principles. In particular the it ambiguity has been argued that the Principle of Relativity only licenses a weak form of the Left Invariance Principle, committed just to invariance of the shape of the light wave not also the magnitude of the light velocity, but it turns out that the Lorentz transformations can still be derived without invoking the stronger form of the Left Invariance Principle (see bibliography). Returning to Einstein's own derivation.

Einstein argued for (2) on the ~~the~~ basis
consider the propagation of a spherical
light wave centred on the origin of the
K- and K' frames at the time $t=t'=0$.
He employed two principles:
1. The light constancy principle that the velocity
of light was independent of the motion
of the source
2. The light invariance principle, that the velocity
of light was the same in any direction
as viewed from the K and K' frames,
~~i.e. that if two wave fronts were seen at~~
~~those times they would be seen as~~
~~were moving outwards from the origin~~
~~and had the same velocity~~.

The light constancy principle already follows from
assuming that light propagates in
accordance with Maxwell's equations
as the light invariance principle would follow
from assuming the invariance of Maxwell's
equations. But Einstein did not want to
assume the full validity of Maxwell's equations,
but wanted to base his relativity theory
on ~~less~~ simpler principles that he thought
would involve sensible alterations to Maxwell's
theory that would be engendered by the
quantum theory (see QUANTUM THEORY).
It is easily checked that the light invariance
principle is obeyed by the Lorentz transformation
equations, whereas the Galilean transformations
clearly violate it.
It does turn out that the full set of Maxwell's
equations are covariant under the Lorentz transformations

* Spacetime intervals:

The interval is often defined as s^2, which is also allowed to be zero or negative. We have chosen to define s so that it is real whenever s^2 is positive, i.e. keeping s^2 always positive.

$$s = \sqrt{(t_2-t_1)^2 - \frac{1}{c^2}\left\{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2\right\}}$$

The events are said to be time-like separated, light-like separated or space-like separated according as s is greater than zero, equal to zero or less than zero.

The metric corresponding to this interval is said to have Lorentzian signature, which distinguishes it from a four-dimensional Euclidean space, where the interval is required to be positive definite.

although as we have just mentioned, this was not fitted into Einstein's derivation.

H. Minkowski (1909) gave a geometrical interpretation of the Lorentz transformation in terms of a four-dimensional geometry equipped with an ~~scalar~~ invariant metric, called the four-dimensional interval between two events located at x_1, y_1, z_1, t_1 and x_2, y_2, z_2, t_2 related to the K-frame as

$$S = \sqrt{(t_2-t_1)^2 - \frac{1}{c^2}\{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2\}}$$

$$= \sqrt{\frac{1}{c^2}\{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2\} - (t_2-t_1)^2}$$

* Minkowski

~~$$S = \sqrt{(t_2-t_1)^2 - \frac{1}{c^2}\{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2\}}$$~~

~~$$\sqrt{(t_2-t_1)^2 - \frac{1}{c^2}\{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2\}}$$~~

~~in which the two events are said to be time-like separated~~

~~$$= \sqrt{\frac{1}{c^2}\{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2\} - c^2(t_2-t_1)^2}$$~~

~~$$\sqrt{(t_2-t_1)^2 - \frac{1}{c^2}\{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2\}}$$~~

~~in which the two events are said to be space-like separated.~~

~~$$= \sqrt{(t_2-t_1)^2 = \frac{1}{c^2}\{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2\}}$$~~

~~in which the two events are said to be light-like separated.~~

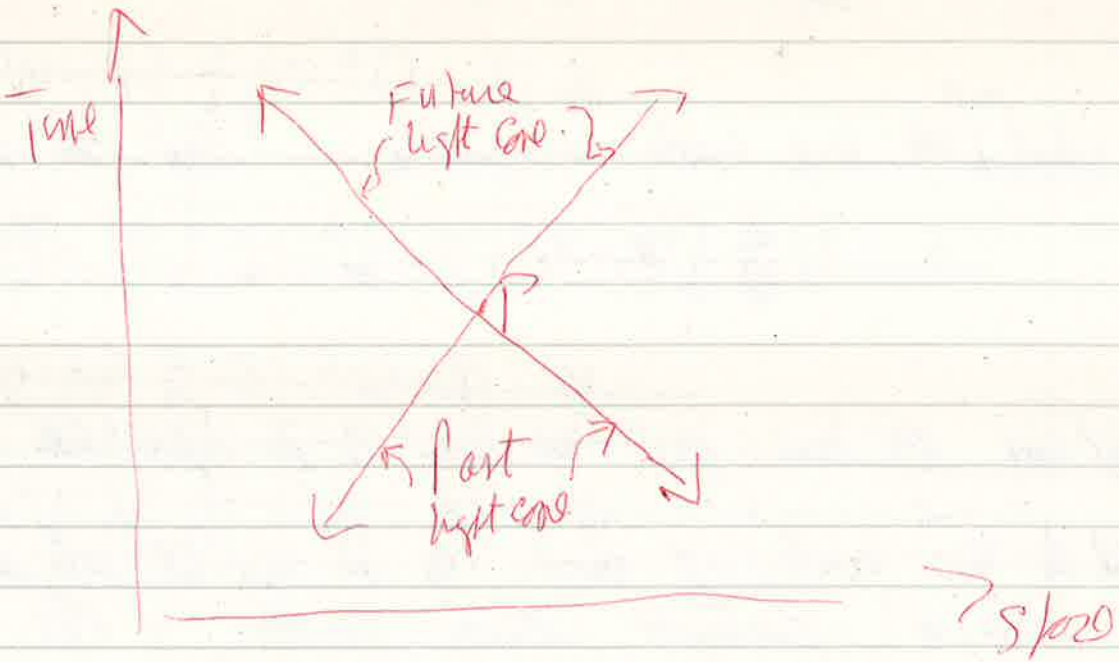
{The interval between two events is the essential under the Lorentz transformation.

$S = 0$ corresponds to the fact that the two events are connectable by a light signal.

Time-like related events are connectable by a signal travelling

* earlier
and adopt a Limiting Velocity Principle
and take

II Earlier The Status of the Limiting Velocity Principle
vis-à-vis the Light Invariance Principle
will be discussed in Section 3 below.



slower than light, while for space-like
separated events, they can not be connected
by signals travelling faster than light.
If the velocity of light is the maximum
velocity that any signal can be propagated
then space-like separated events cannot be connected by
a signal. Timelike separated events can be connected by
physical propagating agent so identified with
causal connection, but then the
causal structure imposed on events in
Newtonian space-time comprises a light-cone
double light-cone constructed at every
point p in space-time the whole of space-time into three classes
of events those that can be causally
influenced by an event at p,
those, as the future-directed light-cone,
the events which can causally affect an
event at p, bounded by the past-directed
light-cone) points on the light-cone
lies connectible with p by light signals,
while the third region is comprises its
events external to the light-cone which
are causally isolated from an event at p.
(see Figure 1)

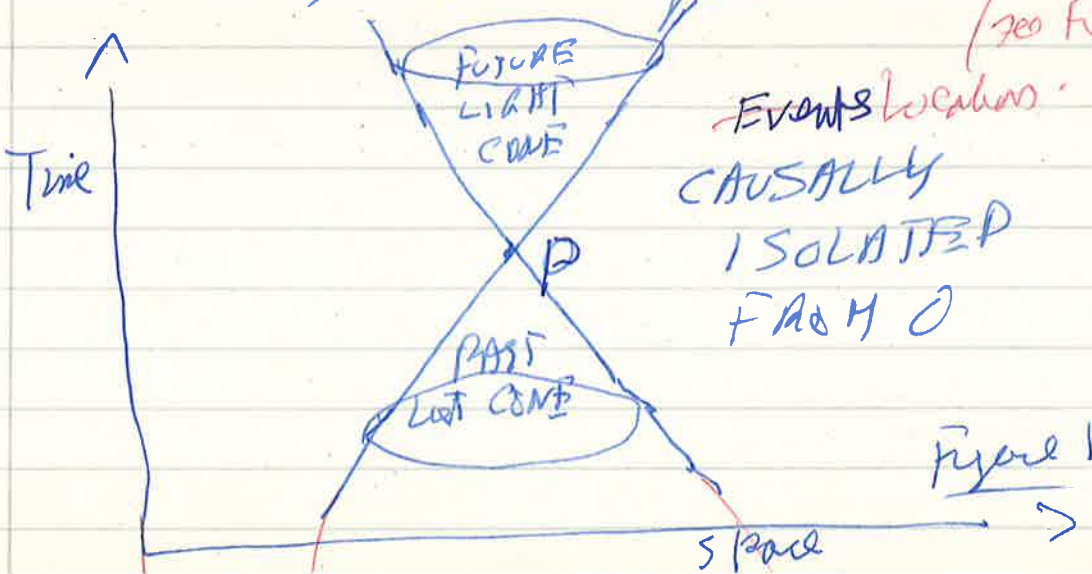


Figure 1

Events
CAUSALLY
ISOLATED
FROM O

No light-like
Trajectories
of relativistic
space-time

Section 3.

As we shall see in a moment, the fact that Rutherford scattering can be employed by physicists such as Niels Bohr and Adolf Smekal as a tool to provide a causal underpinning for Einstein's light quantum

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* (for simplicity we shall ignore the y and 3 spatial dimensions for the time being).

2. The relativity of length, duration and simultaneity.

From the point of view of the Galilean transformations all inertial reference frames agree on which events are simultaneous with one another. The simultaneity relation is an equivalence relation which partitions the collection of all events in spacetime into equivalence classes, which are indexed by the absolute time at which they occur. Absolute time can be understood as the already denoted quotient set associated with the equivalence relation, corresponding to Leibniz's idea of succession, while the equivalence classes themselves, the spatial slices at a given time, correspond to Leibniz's idea of coexistence.

[In the case of the Lorentz transformations however, simultaneity becomes a three-place relation, the third place being filled by the inertial reference frame relative to which the simultaneity of events is being argued. Thus, referring to the transformation equations (2) the locus of events simultaneous with the origin according to the K-frame, i.e. events satisfying the condition $t' = 0$, mapped according to the K-frame, is the locus $t = vx/c^2$, which is a straight line inclined at a slope v/c^2 to the x-axis, i.e. the locus $t = 0$ of events judged to be simultaneous with the origin by the K-frame.* This has immediate consequences for revealing the A-theory of time (SOR TIME) which the

Inverter

(see DETERMINISM).

special theory of relativity. For the A-theorists the present is a determinate simultaneity class of events while dead are co-real. The future comprises unrealized possibilities the past, events that were actual when their occurrence coincided with the present, but are no longer actual now.

~~In not~~ According to special theory if we assume that the relation of co-reality is not frame-dependent, then it is easy to prove that all events in spacetime are co-real. This gives rise to the fact that for any two events A and B in spacetime there always exist a possible event B & C which is simultaneous with A according to one frame of reference K and simultaneous with B according to another frame of reference K'. If A occurs now, then B is co-real with A according to K, so C is now real, while B is co-real with C according to K', so B is also real now. But since B is an arbitrary event it follows that all events in spacetime are real now. Such an argument has also been used to show that special relativity is incorrect with respect to the problem, since the occurrence of all events has a determinate truth value now. ~~Such arguments can of course be rejected if the latter argument can be rejected~~ by deterministic determinateness from determinism. The proper argument shows, however, that special relativity is incorrect with a unique frame-independent notion of becoming.

and hard force is fundamentally opposed to the reordering intuition of the S.-theorist which requires a uniquely determinable present.

[We turn now to the relativity of temporal duration.

Reverting to the transformation equations (2) it follows, remembering that a time interval τ' as measured by a clock at rest at the origin of K is measured as an interval τ by the frame K, then τ and τ' are related by

$$\tau = \frac{1}{\sqrt{1-v^2/c^2}} \cdot \tau' \qquad (3)$$

So the moving clock appears to suffer a dilatation of its time-scale in accordance with equation (3).

But it also follows from (2) that clocks stationary in the K / K-frame, suffer the same dilatation in time-scale as assessed relative to the K'-frame.

This reciprocity of clock dilatations I shall refer to as the clock paradox of special relativity. How can two clocks each run slow as assessed by each other?

It has been claimed that the clock paradox shows special relativity to be a logically inconsistent theory!

But the resolution is simple when proper account is taken of the relativity of simultaneity. Each clock is not being assessed by the other clock in isolation, but only in connection with the relevant standard of simultaneity, which changes accordingly

to that clock reference frame is then used to assess the rate of the clock.

[The clock paradox is closely related to another famous paradox in special relativity to the Twin paradox. Here we consider two twins (biological clocks!) one of whom remains at rest in the K- frame the other moves away and then returns to meet up again with the stationary twin. On comparing clock readings (biological ages) the stay-at-home twin finds a greater lapse of time than the travelling twin. Use follows from the time dilation formula of equation (3).

But from the viewpoint of time dilation used to note follows that according to the stationary frame as that the Traveller is at rest, ~~surely~~ wrong the elapsed stay-at-home twin the younger, when they meet up again? ?

Moreover ~~Dopsin~~ a logical contradiction seems to threaten.

[The resolution of the twin paradox arises from noting that there is no single inertial reference frame relative to which the travelling twin is always at rest. But the inertial rest only holds relative inertial frames. Does this mean that the resolution of the twin paradox requires consideration of accelerated (i.e. non-inertial) reference frames for a complete elucidation, as has been widely claimed in the literature? We shall return to this matter in Section (3).

H Emerton The attempt to use Fortes's theory to motivate a general thesis of the relativity of local particulars to their global context as in the philosophy of Alfred Whitehead is thus seen to be misguided (see WHITEHEAD)

d according to the interpretation of the theory we are adopting in this section (but for a quite different answer see Section 3)

Just as temporal duration becomes a relative notion in special relativity the same is also true of spatial lengths. In place of the distance formula (3) we have it is easy to derive a corresponding contraction formula for lengths

$$L = \sqrt{1 - v^2/c^2} \cdot L' \qquad --(4)$$

where L' is the length of a moving rod as observed by K', and L the length measured by K.

But the relativistic relativization of simultaneity, temporal duration and spatial length does not mean that all space-temporal graphs are relativized to frames. The central idea in special relativity is that there is a unique velocity whose magnitude is the same as observed by all inertial reference frames. In its generalized interpretation by Minkowski, the invariant notion in the geometry is the interval s, and relativity theory would be viewed more accurately, be called the covariance theory of the interval i.e. the theory of transformations under which the interval is invariant. #

Part

Do clocks tell us often asked, do clocks really dilate, or rods really contract when they move, and do they only appear to do so? The answer * is that if the behavior of clocks and rods is governed by the laws of relativistic mechanics, then it is a consequence of these laws, that

clocks and rods contract, as a result of the detailed mechanics of what happens when they are set in motion. There is thus a harmonious correlation between relativistic kinematics and relativistic ~~dynamics~~ mechanics. But at a deeper level we may ask, are the many rods and clocks measuring relativized features of space and time, or is special relativity a theory about space and time per se, or is it a theory about how measuring instruments and rods and clocks behave due to motion, and what, if anything, does this tell us about our concepts of space and time at all? In particular does the invariance of the speed of light as measured by different observers in relative motion to each other arise as a mere artifact of compensating effects in the instruments that result us from ever being able to know our state of absolute motion? The theory of space-time relativity rejects such conventionalist and moves directly to new metaphysical theses about the nature of space and time. The compensatory theories were espoused by H. A. Lorentz and H. Poincaré, and it is they whose theories Einstein rejected, resolutely rejected as opposed to Einstein. But was Einstein's relativity in the sense we have been developing in these readings? In the next section a quite different outlook on the theory will be described. The pages of which can be traced to Einstein's 1905 paper

3. Causality and the Conventionality of Simultaneity

Einstein recognized that the light postulate principle (introduced in Section 1) could not be directly verified by measurement. With a single clock it would be possible to measure the average two-way velocity of light by emitting a light signal at location (spatial location A (marked as event a in figure 2) reflecting it at location B (event 2) and noting the time of event a'' when the signal returns to A. If d is the spatial separation of A and B then the average two-way speed of light would be equal to $2d/(t_{a''} - t_a)$ where the time interval $t_{a''} - t_a$ between events a and a'' is measured by a single clock stationary at A throughout the experiment.

But how could we measure the one-way speed of light from A to B or from B to A? This would require measuring (recording) the time of event 2 by a clock located at B. But this means we would first of all have to synchronize the clocks at A and B. Einstein argued that this was done by defining t_2 to be simultaneous with the event a'' mid-way in time, according to the clock at A, between t_a and $t_{a''}$. Consequently to assume that light propagates with the same speed from A to B or from B to A. But Einstein stated that this is a matter of definition, the one-way speed cannot be measured independently of a decision as how to synchronize spatially separated clocks.

[The line of argument was further developed by Hans Reichenbach who showed that no causal set

$$t_2 = t_a + \varepsilon (t_a'' - t_a) \qquad (5)$$

where the parameter ε has any value between zero and one (the choice being a matter of convention) will function correctly, provided to the choice $\varepsilon = 1/2$.

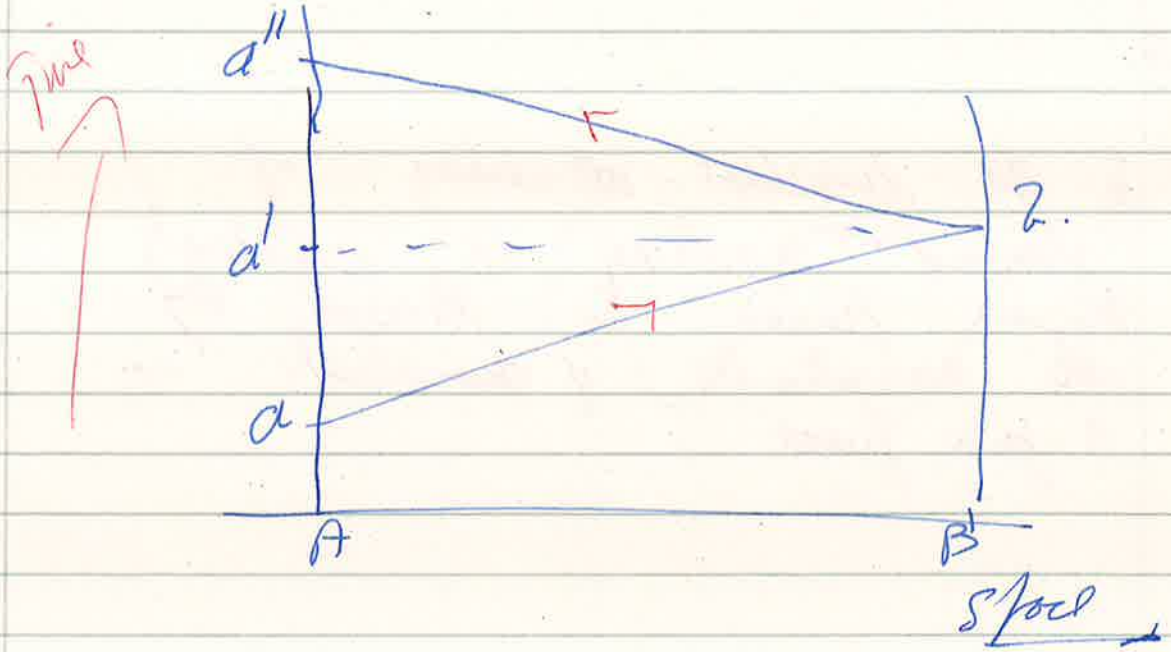

Figure 2

What Reichenbach is claiming then is that t_2 can be defined as appropriate for any event in the closed interval $[a, a'']$. The Conventionality thesis is based, not on the invariance of the light speed, but regarding on the principle that light is the fastest possible signal, thus permitting us [illegible] a faster signal to [illegible] define the conventional solution $[a, a'']$ of events at A associated simultaneously with the event 2 at B

Evaluation

& the descendant attributes of standard Euclidean ordinary in relativity Moving power is licensed by the conventional- / formulations. in a just para

Adolf Grünbaum proposed to make precise
different by defining a concept of
topological simultaneity which relates
any two events not connectible by
a causal signal.
Thus any two events at space-like
separation are topologically simultaneous.
Grünbaum claimed that any more
precise identification of distant simultaneity
~~has~~ coined here no ontological status,
since it could not be stated in
purely causal terms.
It is easy to show that any
two events which are topologically
simultaneous can be made naturally
simultaneous relative to the Einstein
convention applied in a suitably
moving reference frame.
In other words, the range of events
such as $\{a, a''\}$ in Figure 2 that are
topologically simultaneous with a distant events
(such as λ in Figure 2) in a given frame
of reference is exactly the same as the
range of events that are declared naturally
simultaneous with the distant events in different
frames of reference, according to the relativity of
simultaneity (which, of course, follows from the
Einstein convention).
By exploiting the conventionality of simultaneity
in a given frame, we can actually alleviate
the relativity of simultaneity to spatial distance
and length as between different frames!
So, from the point of view, these effects are
not 'real', as opposed to 'effects such as

~~Apart from efforts~~ to the ~~organized~~ of the average two-way road a light which do not depend on any ~~simultaneity~~ convention.

{ The conventionality thesis has attracted considerable discussion in the literature.

Firstly it has been argued that Einstein simultaneity is causally definable. To explain this, we can note that ~~standard~~ Einstein simultaneity in a given reference frame is equivalent to the locus of events which are ~~Minkowski~~ ~~orthogonal~~ in Minkowski geometry to the worldline of the origin of the reference frame. There has been a series of results in the literature showing that Minkowski orthogonality is causally definable in terms (A. Robb, C. Zeeman, D. Malament). So given the worldline of the origin of a reference frame, then Einstein simultaneity in that frame can be defined causally. But arguably the conventionalist response is just to note that relative to a given frame we are free to use the causally defined Einstein simultaneity relative to a moving frame, but as we saw above this is to ~~have~~ ~~as~~ saying non-standard synchrony in the given frame. The same argument can be used to reject the admitted fact that in the event of ~~synchronized~~ slow transport of clocks, fast transport can be used to find out Einstein synchrony. But again the conventionalist claim that in a given

Speaker 1

these are robust limitations on the use of tachyons as a signaling device that arise out of their relativistic properties, and that

Finally we can use clock transport theory
to define relative to a moving frame.
to establish Einstein synchrony relative to
that frame, and hence non-standard
synchrony relative to the original frame.
[It is a remarkable fact that in
four-dimensional Minkowski spacetime the causal
structure determines the complete metric structure
(modulo dilatations), from a point of view
which emphasizes the geometrical structure
of space-time the issue of how we coordinatize
the spacetime can appear trivial, but from the
causal point of view the conventional latitude
in ~~simultaneity~~ simultaneity in a given frame can
be regarded as licensing the discordant
ascriptions of simultaneity relative to moving
frames of the Minkowski interpretation. logically
[The limiting velocity principle is a stronger
principle than the light invariance principle.
Indeed the latter is consistent with the
possibility of tachyons, particles which always
travel faster than light. However it has
been argued by Redhead that sensible
emendations of Reichenbach's causal criterion
for simultaneity can still yield ~~this~~ some
notion of topological simultaneity even in
the presence of tachyons.
[Finally we may note that the conventionalist
approach can be usefully applied to the
twin paradox, avoiding the use of accelerated
reference frames. (~~For further details see the bibliography~~
~~(see Redhead (1990))~~. (See Redhead (1990)).

(Kara)

Einstein Manner

[* + local inertial reference frames can be introduced relative to which gravitational effects can be (locally) eliminated and Einstein assumed a generalized Equivalence Principle (see Schrödinger) according to which all physical phenomena reduce to their SR descriptions relative to such frames.

4. General Covariance

We turn now to the development of the general theory of relativity (GR). Historically Einstein believed this to be an extension of the relativity principle from inertial frames to arbitrary reference frames. However subsequent analysis has shown that GR is a quite different sort of theory from SR as we shall now attempt to explain.

GR considers space-time as a semi-Riemannian manifold. By a manifold we mean that space-time can be coordinatized by quadruples of real numbers and this can be carried out in many different ways, different possible coordinates being related by smooth mathematical transformations. This generality of description replaces the coordinate afforded by inertial reference frames and their relationship via the linear transformations of the Lorentz group type. Just as in Minkowski space-time there is defined an infinitesimal interval between events defined by a metric with a Lorentzian signature, so in G.R. we assume space-time to be equipped with a variable metric, but still with the property that by a suitable smooth transformation the metric can be reduced to the Lorentzian form at a given space-time point, but in general this cannot be achieved over a region of space-time, only at a single point. As first shown by Riemann this corresponds to the fact that the Manifold is curved as opposed to flat over that region.

Einstein's field equations (see Introduction) are of the form $G = \kappa T$ where G is the Einstein tensor

write symbolically in the form

$$G = \kappa T \qquad (6)$$

where G is a quantity related to the curvature of space-time and T represents the distribution of matter and electromagnetic energy throughout space-time. κ is a ~~con~~ fundamental constant representing the coupling of the ~~gravitational effects~~ geometry to the distribution of mass/energy.

The fact that the field equations can be written in so-called covariant form using any system of coordinates in space-time ~~is~~ was called by Einstein the Principle of General Covariance. But the fact that we can describe physical laws in many different ways does not of itself have any physical content, as was pointed out by E. Kretschmann in 1917. In no sense is the Principle of General Covariance a generalization of the Principle of Invariance that arises in SR.

[Weinert] Clarification of this issue was introduced by J. Anderson in 1967 - who expressed covariance and invariance principles in terms of active point transformations on the space-time manifold M instead of the passive coordinate transformations. Smooth point transformations are known to mathematicians as diffeomorphisms. Coordinate transformations induce diffeomorphisms according to the rule that corresponding points have the same numerical coordinates in the two coordinate systems. Geometric objects

* Remark. The diffeomorphisms are said to comprise the covariance group of the space of 1.

field
such O is transformed to a new object field, the drag-along of O, under d a diffeomorphism d, where $d*O$, is evaluated at the transformed point $d(p)$, has the same numerical components with respect to the new coordinate system as does the O evaluated at p, with respect to the old coordinate system.

Anderson partitions the geometric objects O_i into absolute objects A_i which characterize the fixed geometrical space-time background and dynamical objects B_i which characterize the physical contents of space-time.

The actual version of general covariance then states that if $\langle M, A_i, B_i \rangle$ is a model of a space-time theory T then so also is $\langle M, d*A_i, d*B_i \rangle$ for all d.

A space-time symmetry is a diffeomorphism d such that $d*A_i = A_i$, $\forall i$, for all values of i. So then is a space-time theory in which the Riemannian metric is the absolute object and the space-time symmetries are the Lorentz transformations that maintain the invariance of the metric. But in GR there are no absolute objects, the geometrical structure characterized by the metric is itself a dynamical object, and the separate group of diffeomorphisms become space-time symmetries, so the space-time symmetry group and the covariance group are formally identical but the symmetry is no longer has physical content just because it provides no restriction on the covariance group.

Question

and the fact that the geometric structures (curvature, affine connection, etc) are all specifiable in terms of a metric.

[Bronson failed to provide a precise mathematical definition of absolute object, although the intuitive meaning seems clear enough. This technical problem has still not been given a satisfactory solution despite a notable attempt by Friedman (1983) to clarify Bronson's ideas.

[Several comments on the distinctive feature of GR. Indeed Newtonian gravitational theory can also be expressed in generally covariant form (see Malament). The distinctive feature of GR is the dynamic interplay between the geometrical structure of the space-time and the distribution of mass and energy in the space-time. by Einstein

[Solutions of the Einstein field equations yield the ~~the~~ do tell us how the geometric structure (the metric and associated curvature) vary throughout space-time. Einstein originally expressed the field equations with a principle stating that a test-particle introduced into the space-time would follow a geodesic path defined by the metric field. So, in a crisp slogan, a test-particle according to Einstein moves in a straight line in a curved space-time whereas it would move in a curved line in a flat space-time.

[GR The metric ~~test~~ does "double duty", defining both the local space-time geometry and also ~~specifying~~ describing the effective gravitational field acting on a test-particle.

[It was later shown that the geodesic equation

x what we refer to as a 'hole'.

Substantivalism

5. Determinism and ~~Indeterminism~~ in General Relativity

We shall now discuss a number of arguments concerned with the question of whether GR is consistent with a substantival interpretation of space-time or with a relationist interpretation.

[We begin with an ingenious argument that if we assume determinism then GR is inconsistent with a substantivalist conception of space-time points.]

Suppose $\langle M, g, T \rangle$ is a model of GR with metric g and matter/energy distribution T. M is the space-time manifold whose points are treated by the substantivalist as 'individuals' to which properties, the values of the g and T-fields, can be attributed.

Then according to general covariance, for an arbitrary diffeomorphism d, $\langle M, d*g, d*T \rangle$ is also a model.

Now choose (under the substantivalist interpretation) an arbitrary hole in space-time, i.e. a closed region throughout which T vanishes, and take for d a diffeomorphism such that it is the identity map everywhere outside the hole, but unequal to the identity inside the hole. The original model and the new model produced by dragging inside the hole with such a so-called hole diffeomorphism are distinct space-times, in the

sense that inside the hole the same manifold points are associated with different values of the metric field and momenta
But the two models agree anywhere outside the hole so what has been shown is that there can be no physical spacetime inside the hole that is controlled by the state of affairs outside the hole, so determinism in the weakest possible sense is violated.

[Notice that the argument fails in SR, where the symmetry group is sufficiently restricted that hole diffeomorphisms which are also symmetries of the Minkowski metric cannot be constructed.

[In standard modern textbooks of GR spacetimes are only specified up to diffeomorphism or the models that differ only by related spacetimes are empirically equivalent. This of course is true but does not help the thoroughgoing substantivalist who does not believe that empirical equivalence implies ontological equivalence.
The moving point is that the failure of determinism demonstrated is a pervasive feature of GR spacetimes and does not depend on the detailed physics of field equations, equations of motion, &c.

[There has been much discussion of the 'hole' argument in the philosophical literature.

Possible responses include just accepting the failure of determinism as a metaphysical discovery about GR space-times, a jumping up to a stricter version of substantivalism in which the identity of space-time points is independent of properties instantiated at those points, and allowing the identity of space-time points to follow the dragged metric field (metrical essentialism).

Another approach is to resort to counterpart theory and argue that the dragged model refers to the same parts of space-time as the original model, but just to counterparts of those parts. The counterpart-based relations being formulated so as to rescue a definition of determinism that does not fall foul of the hole argument.

No completely satisfactory solution of the problem has been proposed by the substantivalists, but as we shall argue in the next section this does not mean that GR should be interpreted relationally i.e. that space-time just expresses relational properties of the contents of space-time as encoded in the $\mathcal{T}$-field.

6. Mach's Principle

In GR we can raise the question, What determines the local inertial reference frames relative to which the laws of SR hold, according to its generalized Equivalence Principle? The local inertial frames are themselves determined by the g-field, so we can rephrase the question as what determines the g-field? Looking at Einstein's field equations the possible answer suggests itself that it is the T-field i.e. that matter (or other forms of energy) are the source of the g-field. mass-energy. This would suggest that ~~mass-energy~~ is the ultimate origin of inertia rather than space-time itself. If correct this would demonstrate the implementation in GR of the principle enunciated by Ernst Mach in his critique of classical mechanics at the end of the Nineteenth century, that inertial effects arise from motion relative to the average matter in the whole universe. If the matter were removed, inertia would disappear since there would be no chemical ~~absolute~~ Newtonian space serving the purpose of the contents of the Space. Einstein certainly anticipated that GR would support a relativistic conception of space-time and that Mach's Principle would be a crucial feature

of his theory: This turned out not to be the case, for a number of reasons we shall now try to explain.

[Firstly we should note that in the field equations the specification of the T-field on the right-hand side of the equation already involves the g-field, so the T-field cannot be regarded as an independent input to the equations, independent that is of the spacetime geometry. Secondly, the solution of the field equations involve boundary conditions, which again have to be specified in terms of the behaviour of the g-field (Einstein tried to circumvent this difficulty by requiring the universe to be spatially closed, thus obviating the need to specify boundary conditions at spatial infinity). Thirdly, it is possible to define the rotation of the matter field with respect to the local inertial frames specified by the g-field. So it makes sense in GR to talk of matter rotating relative to space-time, rather than with respect to the rest of the material contents of space-time. A real example is the rotating universe discovered by Kurt Gödel (see SPACETIME). Fourthly, we can ask what happens if we set T = 0 throughout space-time. In the spirit of Mach's principle the vacuum equations G = 0 should admit of no solutions. However, many non-flat matter-free solutions of the field equations exist, in which, informally speaking, gravitational energy

Exercise

* on the Einstein field equations

provides the source of the curvature (note that * gravitational energy is not included in the T-field). One concludes that GR is quite inimical to a relational conception of space-time. For pure relationists may indeed envisage space-time as the result of a logical categories construct and regard matter (and electromagnetic) as reducible to certain geometrical features of the space-time. We shall examine this possibility in the final section.

G. K. Ranisch (1925)

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7. Supersubstantivalism

Supersubstantivalism is the view that space-time is not just a substance but indeed the only substance. What we regard as matter is just a region of endowed with certain sorts of geometrical structure, and other candidates for physical reality, such as electromagnetic fields are also to be given a geometrical interpretation. Precursors of this view include Isaac Newton, who interpreted matter in terms of impenetrability imposed on space and Roger Boscovich and Michael Faraday who regarded material atoms as centres of active force, rather than passive material substantial bodies. William Kingdon Clifford applied Riemann's theory of curved manifolds to suggest a geometrical interpretation of matter in terms of local regions of curvature imposed on physical space. Einstein in his own later work sought geometrical explanation of electromagnetic forces in terms of additional geometrical structure such as asymmetric connections or higher dimensions of the space-time manifold.

But in terms of orthodox GR a theory known as geometrodynamics was officially espoused by John Wheeler and his associates. This explored an identification of the electromagnetic field with aspects of the four-dimensional geometry due originally to G. Reimann, electric charges being 'associated with moral

Enzaked

While exchanged mgrs matter was identified with local concentrations of enriched in the style of Clifford.

(~~so-called worm-holes~~)

small-scale topology in the spacetime structure, technically geometrodynamics foreordered in the 1960's due to the pervasive existence of singularities in GR the solutions of the C&D field equations, and problems associated with formulating the initial value problem for the time-evolution of the metric. Nevertheless the spirit of important values has continued to exercise a strong lead on further developments in theoretical physics associated with the programme of quantizing gravitation and unifying it with the other fundamental interactions of electromagnetism and modern physics in the general framework of a geometrically interpreted quantum field theory (see QUANTUM FIELD THEORY).

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Reorder as shown 1-6-

See also: SPACE 4, TIME 6, SPACETIME 5, ELECTRODYNAMICS 3, COSMOLOGY 1, ~~EINSTEIN~~ EINSTEIN 2

References and Further Reading